

The Theory of Superfluidity of ^4He

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I present here a microscopic theory for the superfluidity of ^4He (He II) derived from experiments, and answer its essential questions. With a "momenton" model, the superfluid is shown to feature as a "harmonic superfluid". In which a new bonding type, the "superfluid bond", is formed. Its activation causes the anomalous thermal excitation, the large excess of specific heat, etc. The superfluidity mechanism is recognized being connected to a quantum confinement effect. The theory predicts the basic properties of He II in overall agreements with experiments. The series novel concepts evolved here reverse the current perceptions of ^4He , give significant impact to the understandings of other superfluids.

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"What is physics, physics is what one observes from experiments . . ." — By a condensed matter theorist.

^4He , being small, light and inert, occupies hence a special position among the chemical elements. The striking manifestation of these extremes in the bulk form is that ^4He does not solidify down to whatever low temperature under ambient pressure, transforms instead at $2.17\text{ }^\circ\text{K}$ from a classical liquid (the normal liquid, or, He I) to the superfluidity phase (the superfluid, or, He II.) The very characteristic property of the superfluid is its absence of friction in some kinematics processes and is also why the name "superfluidity" is given. Enormous amounts of experiments have been made over the past 80 years to learn about the unusual fluid of He II. To formulate a satisfactory theory for it, however, has presented as an intriguing challenge. The difficulty of the problem is three-fold: He II is a liquid; on top of that it is a quantum liquid; its ability to flow without dissipation is out of normal place among the great majority of condensed matters. He II exemplifies one of the extreme cases that endure the validity of the normal physics laws. Briefly speaking, it is generally agreed that there is so far no satisfactory theory for the ^4He superfluidity. Comprehensive reviews on He II are given e.g. in [1]- [4].

In this letter, I present the backbones of the microscopic theory of the superfluidity of ^4He . The complementary details and extended contents of the theory are presented in a much elaborate paper [5]. The theory has been formulated from iterative contrasts between what is predicted by an assumed "theory" (solutions of equations of motion based on a microscopic model construction) and what is observed by the experiments; and contrasts between what is predicted for one property and its impli-

cation to the others; and followed by modifications of the "theory". The approach follows treating the superfluid as a "modified solid".

The formulation of the theory based on experimental observations

a. Thermal excitations between $0 \sim 0.6\text{ }^\circ\text{K}$, phonons
In this T region, He II is observed in various experiments (see e.g. [1]) to be virtually a (pure) superfluid. In the meantime, experiments point to that He II in this region, hence the (pure) superfluid, predominantly features as assemblies of harmonic oscillators about relatively fixed sites, assuming some intermediate range of ordering, the associated thermal excitations being creations of longitudinal single phonons; I thus term the superfluid the "harmonic superfluid". That is consistently pointed to by: (i) neutron scattering intensity for a given momentum transfer, q , presents [6] a sharp peak centred at a finite frequency and the excitation dispersion resembles [5] that of a harmonic crystal except at $q_b \simeq 1.93\text{ }1/\text{\AA}$, see Fig 1 (data from [7]); (ii) the heat capacity has a Debye T^3 behaviour [5] (data from [8]); (iii) a velocity of first sound $c_1 \simeq 239\text{ m/sec}$ is consistently given [5] by (i) and (ii) as well as by pulse transmission measurements (e.g., [9].) Subsequent to the above, I infer that any of the relevant properties of He II ought to normally result from the *involvement* of the single phonon excitations.

b. The superfluid bond The harmonic superfluid character and its associated suppression of atom diffusion into localized vibrations effectively imply that the bonding of He atoms in the superfluidity state has undergone a qualitative change and is larger compared to in He I. I term the new bonding type as "superfluid bond", defined as the negative total binding energy per He atom (U_{os}), and denoted as $E_s(q) (\simeq -U_{os})$, with $E_b (= E_s(q_b))$ referring to the zero phonon superfluid bond at q_b , cf Fig 1. Quantitatively, $E_s \approx E_b$, as well shall see. Further relevant experimental indications include: (i) a large excess of specific heat of a λ profile presents between about $0.6 (\sim 1)$ to $2.17\text{ }^\circ\text{K}$, see Fig 2 (data from [10], [11];) (ii) an anomalous sharp inelastic neutron scattering peak presents [6] at q_b of an energy transfer of $\Delta_s \simeq 8.6\text{ }^\circ\text{K}/\text{atom}$, cf Fig 1; (iii) thermodynamic measurement [12] yields the ground state cohesion energy, $U_{os} \simeq -7.2\text{ }^\circ\text{K}/\text{atom}$; (iv) diffusion of a He ion in the superfluid [13] involves an activation energy of $8.1 \sim 8.6\text{ }^\circ\text{K}/\text{atom}$. (i)-(iv) consistently point to the presence of a superfluid bond, $E_b (\simeq -U_{os}) = \Delta_s - E_v \simeq 7.2\text{ }^\circ\text{K}/\text{atom}$, E_v

being a small energy needed for site creation. It follows that, between $0.6 (\sim 1) \sim 2.17$ °K, He II is a co-existent of the superfluid and the normal fluid, and a large excess heat is required for heating up and is consumed mainly to convert the super- to the normal fluid via activation of the superfluid bond.

c. The momenton wave and collective phonons Despite of a and b , He II is a liquid. To reflect this solid-liquid duality but with *solid* being much emphasised, I introduce a "momenton" description of the superfluid. A "momenton" is defined as an aggregation of a macroscopically large number of He atoms which rarely diffuse individually but primarily oscillate about sites fixed on the centre of mass coordinates of their momenton. Superposed to the *independent* oscillations (the single phonon events) are slow *in-phase* and *simultaneous* vibrations of all atoms following the momenton, which give rise to collective phonons whose energy contribution though is usually negligible [5]. The superfluid atoms also translate in such a collective fashion. At T ($0.6 \sim 2.17$ °K) where the two fluids co-exist, they each aggregates in large islands of regions, rather than intermingles on an atomic scale. Relevant experimental indications include: a slow (second) sound wave can propagate in He II and its pulse broadens as T falls (e.g. [14]); thermal conductivity in He II is exceptionally large, and not defined by temperature gradient.

The equations of motion

Based on b and c , I can readily simplify the complex superfluid atom motion by coordinates decomposition, and in the semi-classical forms we have:

$$-\alpha_1 \sum_{j=1,j,j+1} \mathbf{u}_i = m \frac{\partial^2 \mathbf{u}_j}{\partial t^2} \quad (1)$$

$$D \nabla^2 n(\mathbf{R}) - \frac{\partial n(\mathbf{R})}{\partial t} = 0 \quad (2)$$

$$-\alpha_2 \sum_{j=1,j,j+1} \xi_i = M_j \frac{\partial^2 \xi_j}{\partial t^2}, \quad j = 1, 2, \dots, N_M \quad (3)$$

$$F_{12} - \Upsilon = \sum_i^{N_M} M_i \frac{\partial^2 \Xi}{\partial t^2} \quad (4)$$

Their quantum mechanical representations can be similarly decomposed [5]. In above, the He atom absolute position \mathbf{r} is a vector sum of four relative coordinates: $\mathbf{r}(x, y, z) = \mathbf{u} + \mathbf{R} + \boldsymbol{\xi} + \boldsymbol{\Xi}$, as schematically illustrated in Fig 3. Of Eqs (1)-(4), (1) and (3) describe the vibrations of a He atom and a momenton, for their solutions and discussions see [5]. (2) and (4) are discussed below.

a. Excitation of superfluid bond and the total excitation Under thermal equilibrium, Eq (2) describes, in terms of the liquid density $n(\mathbf{R})$, a single atom translation, \mathbf{R} , relative to the momenton coordinates, $\boldsymbol{\Xi}$. It can be thermal diffusion, or atom displacement caused by an

external perturbation such as an incident neutron, or an applied field, etc. These all involve a probability to excite an atom from a bound state of the bonding energy E_b , amounting to the proportion to the Boltzmann factor,

$$P_s(\mathbf{R} \rightarrow \mathbf{R}') \propto D \propto e^{-\frac{\Delta_s}{k_B T}} = e^{-\frac{(E_b + E_v)}{k_B T}} \quad (5)$$

Due to certain lattice ordering in the superfluid, the superfluid bond excitation is dependent on the phonon wavevector, K , as being explicitly shown in the neutron data. And the maximum intensity occurs at $K_b \propto 2\pi/a \simeq 1.93$ 1/Å, i.e., the reciprocal lattice vector (expressed for a cubic lattice.) If it were a solid (of bonding energy: $\sim 10^5$ °K/atom), it would cause neutron an elastic Bragg scattering at K_b . Due to an intermediate bond strength (~ 8.6 °K/atom) which falls below the thermal neutron energy (60 to 1160 °K/atom), the superfluid instead scatters a neutron at K_b with an energy transfer of $\Delta_s = (E_b + E_v) \simeq 8.6$ °K/atom, see Fig 1. For $|K - K_b| > 0$, a neutron begins to excite phonons more readily, thus P_s drops rapidly within a narrow width $\sim \sigma$. So that, for $|K - K_b| > \sigma$, $P_s \simeq 0$ and the full excitation involves primarily phonons: $\mathcal{E}(K) \approx \hbar\omega_{ph}$; for $|K - K_b| < \sigma$,

$$\begin{aligned} \mathcal{E}(K) &= \Delta_s + \hbar\omega_{ph} \approx E_b(K_b) + E_v + \frac{\hbar c_1}{k_B} |K - K_b| \quad (6) \\ &\approx 8.6 + 18.2 |K - 1.93| \quad (\text{°K/atom}) \end{aligned}$$

where $\hbar = h/2\pi$ and k_B being the Planck's and Boltzmann constants; $\hbar\omega_{ph}$ being expressed for the acoustic phonons. The parameterized Eq (6), obtained using values of Δ_s and c_1 given earlier, is as plotted in Fig 1 in the vicinity of K_b (full line-2), and is seen to reproduce the experiment curve here reasonably well.

b Superfluid flow, translation of momentons and atoms For a uniform and steady flow, Eq (4) describes the translation of a momenton or an atom or the flow under an external pressure difference, F_{12} , Υ being the viscous resistance, $\sum_i^{N_M} M_i = M_s$ the flow mass, M_i the mass of the i th of the total N_M momentons. For the case as above, there is, $\langle \mathbf{r} \rangle = \boldsymbol{\Xi}$ ($\langle \rangle$ refers to time average over an infinitesimal $\boldsymbol{\Xi}$ translation.) Provided the translation velocity, v_s , is below a critical value, so that $\Upsilon \equiv 0$, the solution of Eq (4) describes the non-dissipative superfluid translation at the absence of F_{12} :

$$v_s = \frac{\partial \langle \mathbf{r} \rangle}{\partial t} \equiv \text{constant} \quad (7)$$

The quantum mechanical counterpart describing a He atom translation is an *effective* planewave [5],

$$\Psi(\mathbf{r}) = C e^{i(\mathbf{k}_s \cdot \mathbf{r} - \omega t)} \quad (8)$$

where $\mathbf{k}_s = mv_s/\hbar$, m is He atomic mass. k_s and v_s are both *effective* in a sense that an atom is accelerated with an inertia equal to the flow mass, M_s

The mechanism of superfluidity

In the harmonic superfluid, conveyed in the (longitudinal) elastic wave propagation is that of phonons. Normally phonon excitation energy: $\Delta\omega(K_{ph}) \simeq c_1\Delta K_{ph}$ can be arbitrarily small for ΔK_{ph} being small. When in a narrow channel, however, its width d then sets a maximum wavelength λ_{max} for phonons able to propagate in the superfluid, which corresponding to the minimum phonon wavevector and an associated energy gap,

$$(K_{ph})_{min} = \frac{2\pi}{\lambda_{max}} = \frac{2\pi}{d}, \quad \text{and} \quad \Delta_{ph} = \hbar\omega_{ph}((K_{ph})_{min})$$

this being referred as one type of the "quantum confinement effects" (similar effect for free particle like atoms is treated in [5].) It leads to that no lower modes phonons than $(K_{ph})_{min}$ can be excited in the confined superfluid sample here by the flow and wall interaction *This is the cause for superfluidity motion of He II* (at larger d .) This mechanism is pointed to by the two characteristic experimental demonstrations: (i) a He II rotation current persists over the entire experiment duration (12 hrs) [15]. (ii) He II translation exhibits non-dissipative superfluidity only in narrow channels and below a critical velocity, v_c , and $v_c(d)$ increases as d falls, see Fig 4 (data from [1]).

I now establish the first principles criteria for it and evaluate $v_c(d)$. The energy exchange between the flow and the wall is via N_{surf} atoms at the interface, each having the flow inertia M_s ; and hence is via an effective mass: $m_{eff} = M_s/N_{surf} = md/4a$. Upon collisions with the wall, the flow loses its translation momentum (initially k_s, v_s) and energy ($\propto M_s v_s^2/2$), so that in the severest case being stopped ($k'_s = v'_s = 0$.) Being inert, the superfluid atoms can be approximated as not interacting with the wall except for the moments of collisions. The heat thus produced is converted to disordered atom vibrations in the bulk — the creation of phonons, of the energy $\hbar c_1 K_{ph}$ in the case of low excitations. A non-dissipative superfluid translation hence demands the two simultaneous conditions:

$$k_s \leq (K_{ph})_{min}, \quad (10)$$

$$m_{eff}v_s^2/2 \leq \Delta_{ph} = (\hbar c_1 K_{ph})_{min} \quad (11)$$

Using (9) in the more restrictive Eq (11) yields the critical velocity:

$$v_c = \sqrt{\frac{8hc_1a}{m}} \cdot \frac{1}{d} = 2.62 \times 10^{-7} \frac{1}{d \text{ (m)}} \quad (\text{m/sec}) \quad (12)$$

The parameterized Eq (12) is obtained with: $c_1 = 239$ m/sec, and $a = 3.6 \text{ \AA}$ (the He II atomic spacing.) The predicted $v_c(d)$, full line in Fig 4, is seen to agree well with experiments for larger d . For low d , improved $v_c(d)$

(dashed line) has been obtained [5] by treating the He atoms appropriately as free particles.

Based on the microscopic theory outlined above, a statistical thermodynamic description of He II is also given in [5]. Amongst others it predicts the experimental properties, T_λ and C_V (full lines of Fig 1), etc, with overall satisfaction. Some of the results were used in the discussions here, where Ref [5] is referred.

I end this letter with some complementary remarks. (1) The "harmonic superfluid" character implies that the (9)(pure) superfluid is thermally excited to a full extent in the longitudinally modes, no less than any solid. So, (1.a) it is not an assembly of *motion-less* atoms that was thought to lead to superfluidity, (1.b) neither its *reminiscing a harmonic solid alone* explains superfluidity since a solid does not exhibit "superfluidity". (2) It follows, from the substantial atomic bonding, that the superfluid internal viscosity is large, especially compared to that of He I. So the superfluid is not an assembly of *non-interacting atoms that would result in zero internal viscosity*. Particularly, Stoke's theorem hence does not apply to the superfluid and to ensure non-turbulence thus does not require a *total circulation zero*, which being instead quantized [5]. It follows that no *local vortices* present in the superfluid. (3) Based on quantum mechanical solutions, it can be elucidated [5] that the "superfluid bond" results from the quantum He atom mediation among neighbouring sites, and it differs from the known chemical bonds that are mediated via electrons. (4) What eases one to face the observed bulk properties is the recognition of the superfluidity mechanism here, which appears made conclusive by the good agreement between the first principles evaluation of $v_c(d)$ (without any adjustable parameter) and experiments. (5) The same as the bulk phenomena are shown in the theory to be interrelated, quantitative evaluations of the He II properties mainly are based on two common parameters, c_1 or/and Δ_s which being well determined from experiments, and are fully or partially satisfactory. (6) The essential properties of the superfluid of ${}^4\text{He}$ have been explained based on a unified theory ground; and the physics necessitating the depiction of the extreme case of ${}^4\text{He}$ is shown to retain the physics laws that apply to normal matters.

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persistent current experiment.

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FIG. 1. Thermal excitation energy versus phonon momentum K (or neutron momentum transfer q) for the superfluid of ^4He , from neutron diffraction (circles) at 1.12 °K and theory (full lines.) About full line-1 and the dashed and dotted lines see [5].

FIG. 2. Specific heat C_V of superfluid He II, from theory (full lines) and experiments (circles)

FIG. 3. Illustration of relations of the relative coordinates \mathbf{u} , ξ , Ξ and the absolute position \mathbf{r} of a superfluid atom.

FIG. 4. Critical velocity $v_c(d)$ for the superfluid of ^4He , from theory (full and dashed lines) and experiments (circles) at about 1.4 °K.

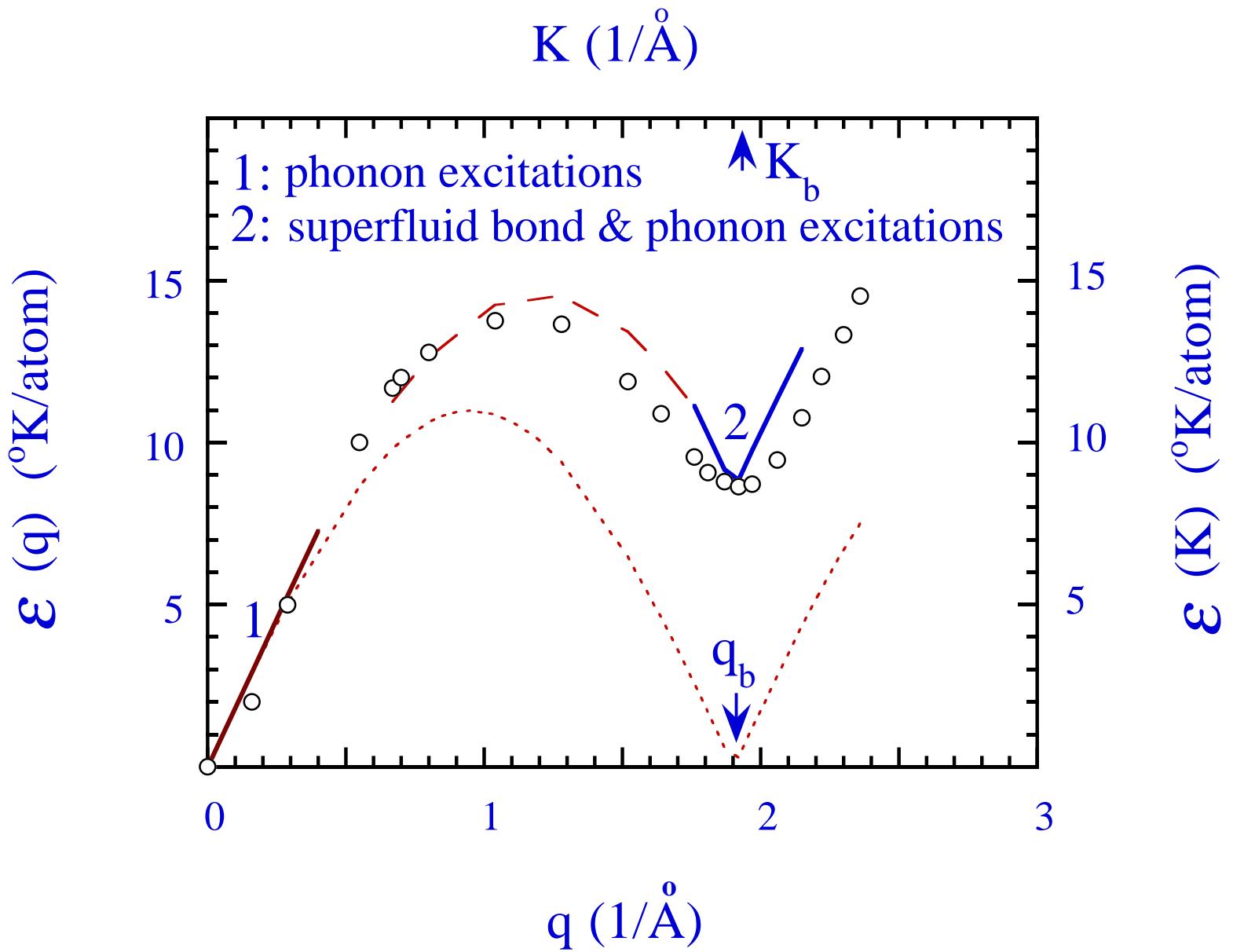


Figure 1 (J X Zheng-Johansson)

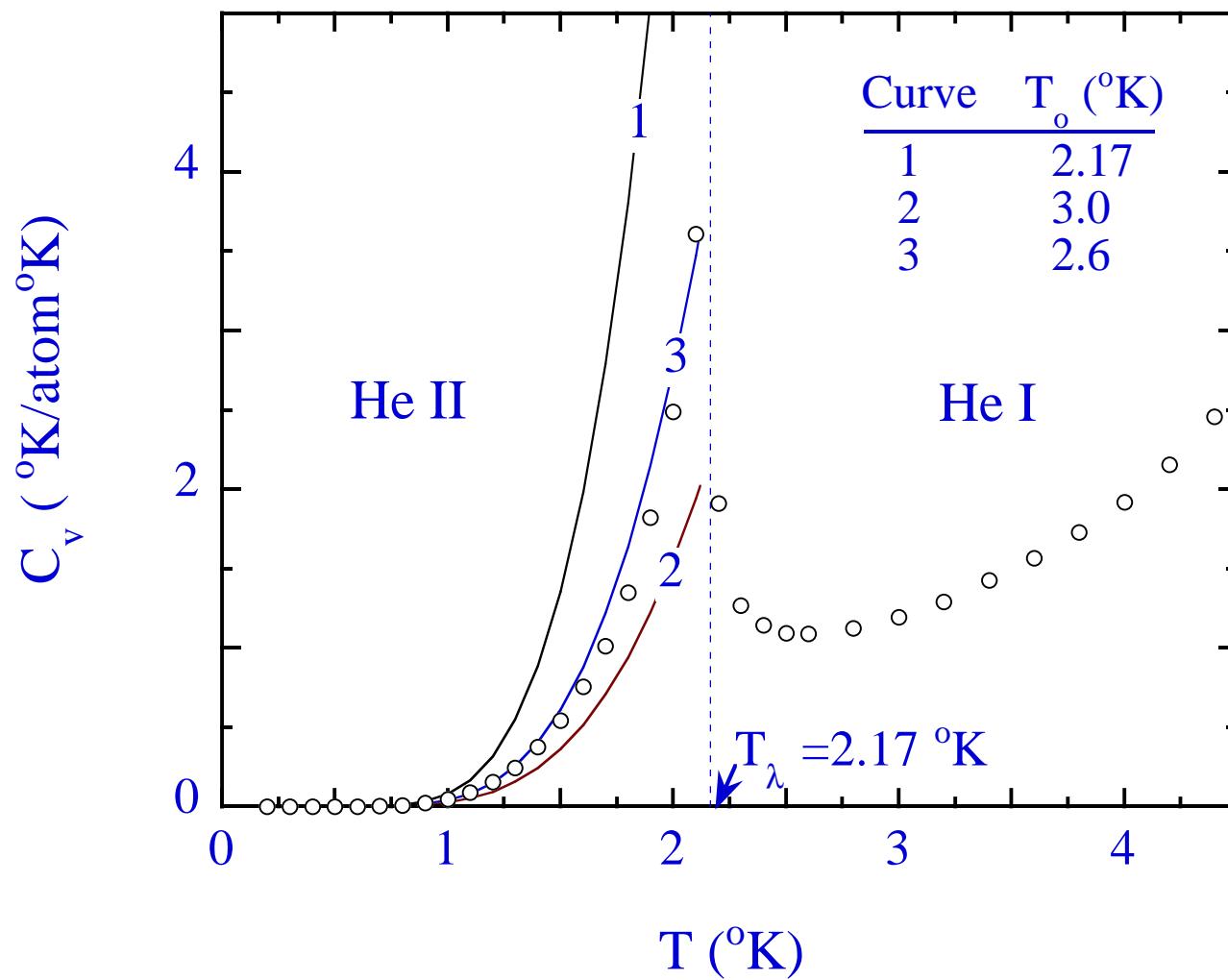


FIG 2 (J X Zheng-Johansson)

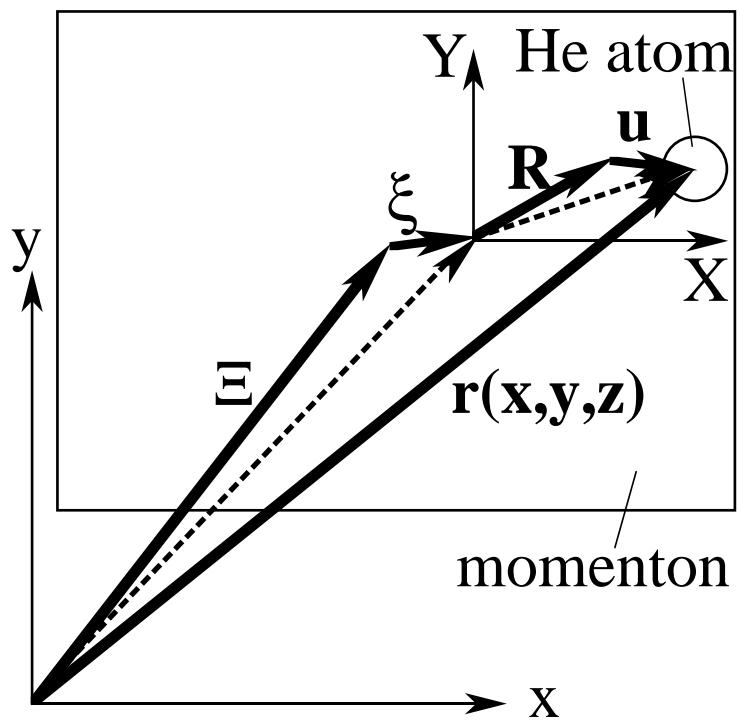


Figure 3 (J X Zheng-Johansson)

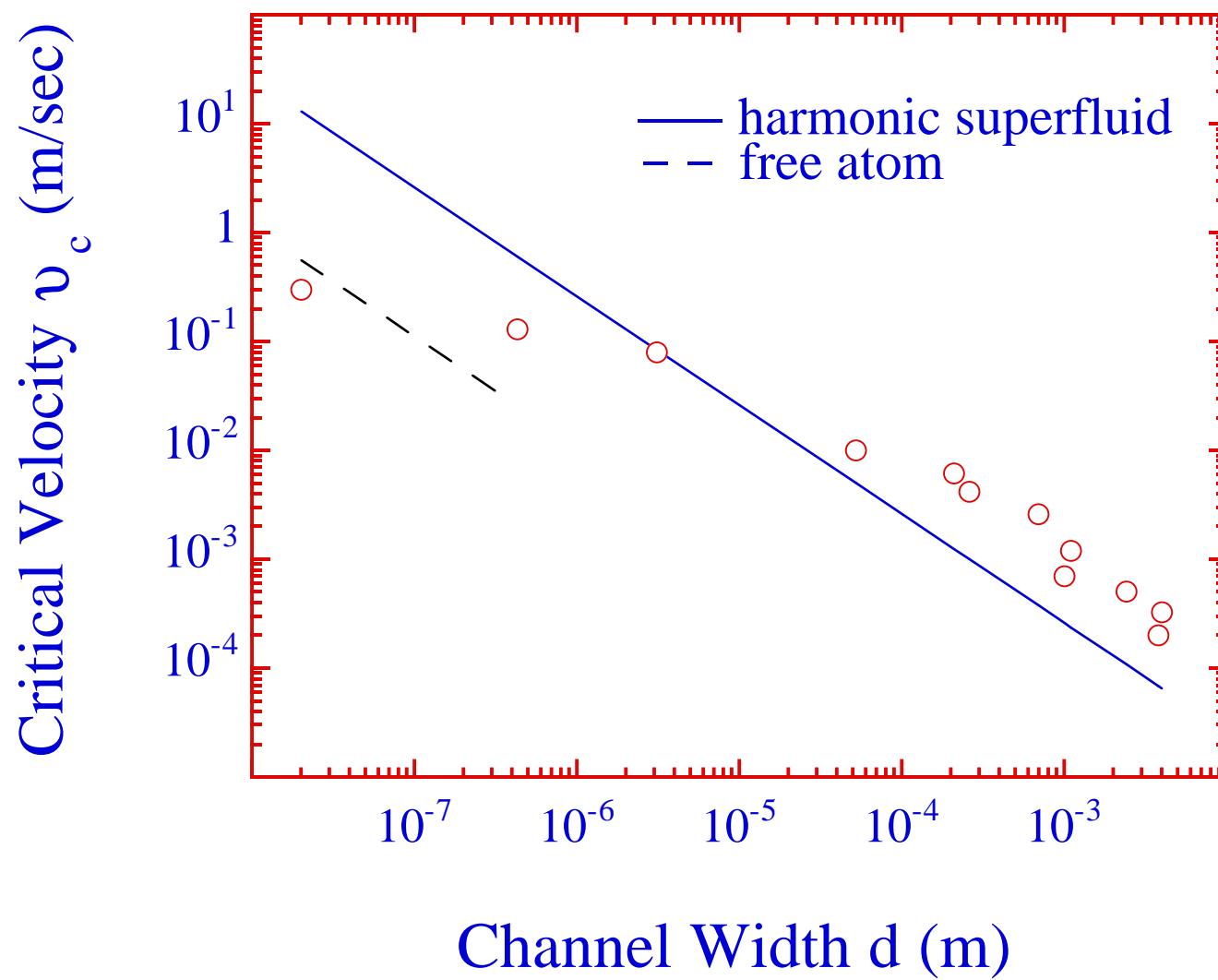


Figure 4 (J X Zheng-Johansson)